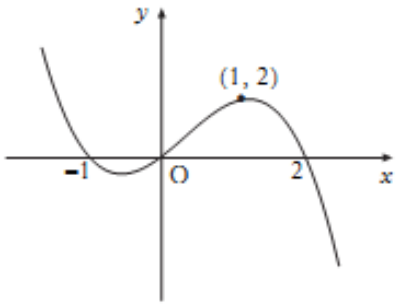
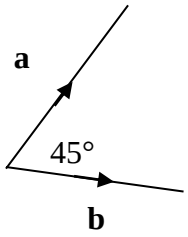
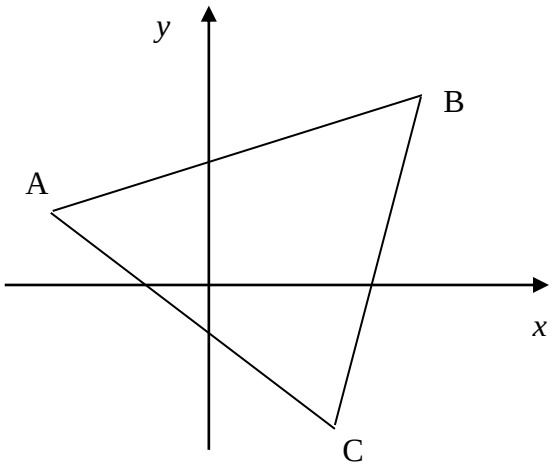
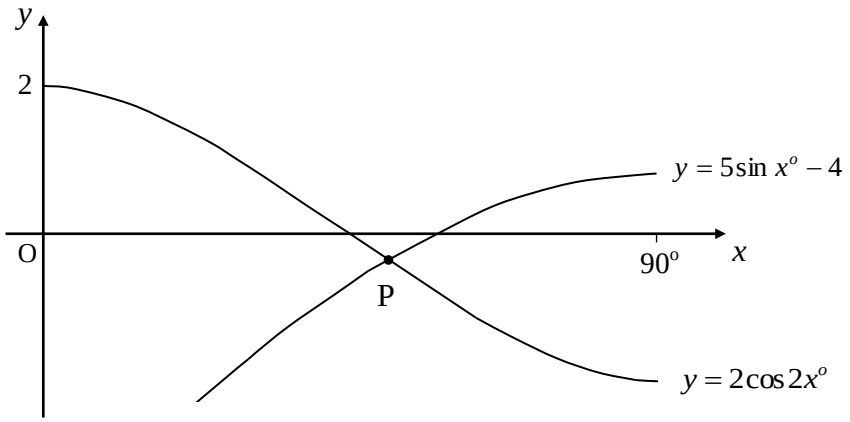
	Higher Prelim Revision 3 – Paper 1 Non-Calculator	**30**
1.	Line AC makes an angle of 120° with the positive direction of the x-axis. Calculate the gradient of the line AC	**3**
2.	Vector a***i*** + b***j*** + ***k*** is perpendicular to both the vectors $2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{j}$ Find the values of a and b	**3**
3.	Two functions, defined on suitable domains, are given as $f(x) = \frac{1}{x} - 4$ and $g(x) = -8x$. Find the value of $g(f(0 \cdot 5))$	
4.	The diagram below shows part of a graph of a cubic function.  This curve passes through the points (-1,0), (0,0), (1,2) and (2,0). Find the equation of this function	**3**
5.	Prove that vectors A(-1, -8, -2), B(2, -5, 4) and D(3, -4, 6) are collinear and state the ratio in which B divides AD	**3**
6.	Express cos$x$ + sin$x$ in the form kcos(x – a), where k > 0 and $0 \le x \le 360^\circ$	**4**
	Hence or otherwise state the range of the function y = cos$x$ + sin$x$	**2**
7.	Given that $f(x) = \sin^2 x$, find the exact value of $f'(\frac{\pi}{3})$	**3**
8.	The diagram shows two vectors **a** and **b**, where |**a**| = 3 and |**b**| = $2\sqrt{2}$ The vectors are inclined at an angle of 45° (a) Evaluate (i) **a** ● **a** (ii) **b** ● **b** (iii) **a** ● **b**  	**3**

9.	Prove that the function $f(x)=\frac{1}{x+1}$ is decreasing for all values of x except x = -1	**4**

Answers

1.	$M = \tan\theta$, $\rightarrow m = \tan 120° \rightarrow m = -\tan 60°$ $m = -\sqrt{3}$
2.	$\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0$ *and* $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0 \rightarrow$ 2a + b + 1 = 0, a + b = 0 $\rightarrow$ **a = -1, b = 1**
3.	$f(0.5)=\frac{1}{0.5}-4=-2,\ g(-2)=-8(-2)=16$ Or $g(\frac{1}{x}-4)=-8\left(\frac{1}{x}-4\right) \rightarrow -8\left(\frac{1}{0.5}-4\right)=16$
4.	$y = kx(x + 1)(x – 2)$ $\rightarrow$ $2 = k(1)(1 + 1)(1 – 2)$, $2 = k(1)(2)(-1)$, $2 = -2k$, ***k* = -1** the equation of the function is ***y* = -*x*(*x* + 1)(*x* – 2)** or ***y* = -*x*³ + *x*² + 2*x***
5.	$\vec{AB}=\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$, $\vec{BD}=\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $\vec{AB} = 3\vec{BD}$ so vectors are parallel. B is a common point so A, B and D are collinear. B divides AD in the **ratio 3:1**
6.	$k\cos(x – a) = k\cos x \cos a – k\sin x \sin a$, $k\cos a = 1$, $k\sin a = 1$, $k = \sqrt{2}$, $a = 45°$ $\sqrt{2}\cos(x-45)\rightarrow$ the range of this function is $-\sqrt{2} \le y \le \sqrt{2}$
7.	$f'(x)=2\sin x\cos x,$ $f'(\frac{\pi}{3})=2\sin\frac{\pi}{3}\cos x\frac{\pi}{3}=\frac{\sqrt{3}}{2}$
8.	**a** ● **a** = $3^2 = 9$, **b** ● **b** = $(2\sqrt{2})^2 = 8$, **a** ● **b** = $3 \times 2\sqrt{2} \times \cos 45° = 6$
9.	$f(x)=(x+1)^{-1},$ the restriction on the domain of $f'(x)$ is $x + 1 \neq 0$, $x \neq -1$ $f'(x)=-1(x+1)^{-2}$ $=-\frac{2}{(x+1)^2}$ for all other values of x, $f'(x) < 0$, so $f(x)$ is always decreasing

	Higher Prelim Revision 3 – Paper 2 Calculator	**45**
1.	A triangle has vertices A (-4, 1), B (12, 3) and C (7, -7)  (a) Find the equation of the median CM (b) Find the equation of the altitude AD (c) Find the coordinates of the point of intersection of the lines CM and AD	**3** **3** **3**
2.	(a) Show that $(x – 2)$ is a factor of $x^3 + 2x^2 – 4x – 8$ and hence fully factorise this polynomial (b) Hence solve $x^3 + 2x^2 – 4x – 8 = 0$ (a) Find the coordinates of the stationary points of the curve with equation $y = x^3 + 2x^2 – 4x – 8$ and hence state their nature (b) Sketch the curve of $y = x^3 + 2x^2 – 4x – 8$, clearly marking the x and y-intercepts and the stationary points	**4** **1** **7** **2**
3.	(a) Express $f(x) = 3x^2 – 12x – 15$ in the form $a(x – p)^2 + q$ (b) Sketch the graph of (i) $y = f(x)$ (ii) $y = 10 - f(x)$	**3** **1** **3**

4.	The rate of decomposition of an acid in a solution obeys the law $C = 50\,e^{-0.024t}$, where C is the concentration, in millilitres per litre, left after t hours. (a) What is the initial concentration of acid ? (b) Determine how long it takes for the concentration to reach 20ml/litre, giving your answer correct to the nearest hour.	**1** **4**
5.	 The diagram shows part of the graphs of $y = 2\cos 2x^o$ and $y = 5\sin x^o - 4$ in the interval $0 \le x \le 90$. Form an equation and solve it to find the coordinates of P	**5**
6.	Evaluate $\int_1^2 \frac{1}{2} + \frac{1}{u^2}\, du$	**5**

Answers

1a	Midpoint of AB is (4,2) $m_{med} = -3$	equation of median is $\boldsymbol{y = -3x + 14}$
1b	$M_{CB} = 2$, $M_{alt} = -\frac{1}{2}$	equation of altitude is $\boldsymbol{y = -\frac{1}{2}x - 1}$
1c	$-\frac{1}{2}x - 1 = -3x + 14$, $x = 6$, $y = -4$	point of intersection is (6,-4)
2a b	• 2 | 1 -2 -4 -8 0 2 8 8 1 4 4 0	No remainder, (x - 2) is a factor $(x - 2)(x^2 + 4x + 4) = (x - 2)(x + 2)(x + 2)$ Solutions are $x = 2$ and $x = -2$

2c

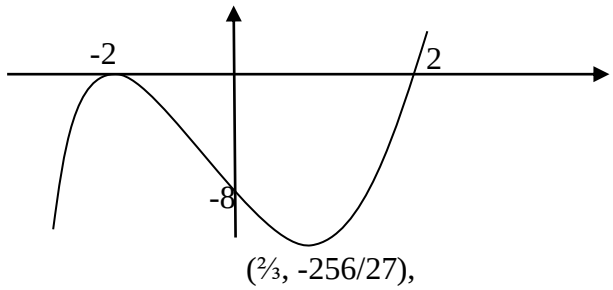
$\frac{dy}{dx} = 3x^2 + 4x - 4 = 0 \quad \rightarrow (3x - 2)(x + 2) = 0 \quad x = ⅔, \quad x = -2$

$y = -256/27, \quad y = 0$

x	-2^-	-2	-2^+	$⅔^-$	$⅔$	$⅔^+$
$\frac{dy}{dx}$	+	0	-	-	0	+

Max at (-2,0), min at (⅔, -256/27),

2d



(⅔, -256/27),

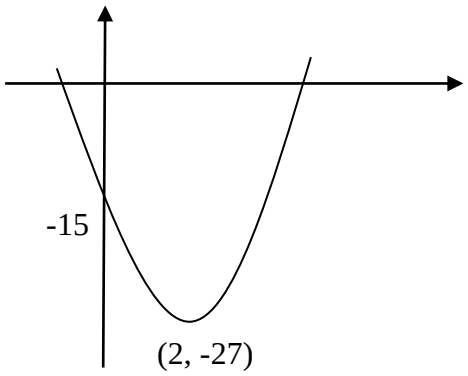
3a

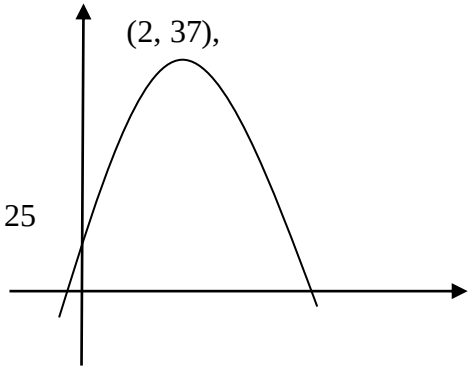
$3x^2 - 12x - 15 = 3[x^2 - 4x] - 15 = [3(x - 2)^2 - 4] - 15 = 3(x - 2)^2 - 27$

3b

Turning point (2,-27) → $y = 10 - f(x)$ Turning point (2,37)

y-intercept (0, -15) y-intercept (0, 25)





4

Initial concentration is 50 ml per litre

$20 = 50\,e^{-0.024t} \rightarrow \frac{2}{5} = e^{-0.024t} \rightarrow \ln(\frac{2}{5}) = -0.024t \rightarrow \frac{\ln(2/5)}{-0.024} = t$ **t = 38 hours**

5.

$\cos 2x = 1 - 2\sin^2 x$ so

$2(1 - 2\sin^2 x) = 5\sin x - 4 \quad \rightarrow \quad 0 = 4\sin^2 x + 5\sin x - 6$

$0 = (4\sin x - 3)(\sin x + 2)$

$\sin x = ¾$ and $\sin x = -2$

$x = 48.6°, 131.4°$ no solutions **P(48.6°, -0.25)**

6.

$\int_1^2 \frac{1}{2} + u^{-2}\, du = \left[\frac{1}{2}u - \frac{1}{u}\right]_1^2 = \left(\frac{1}{2}(2) - \frac{1}{2}\right) - \left(\frac{1}{2}(1) - \frac{1}{1}\right) = 1$